Effect of Parton Momentum Spread on Hadron Elliptic Flow in Relativistic Heavy Ion Collisions

Colin C. Young
Cyclotron Institute, Texas A&M University
Advisor: C. M. Ko

Scaling of Elliptic Flow

- The dependence of the elliptic flow of hadrons produced in a heavy ion collision on their transverse momentum becomes similar if it is divided by the number of quarks in the hadron.
- This scaling is consistent with the quark coalescence model of hadron formation, in which 2 or 3 partons of similar momentum coalesce to form a hadron with 2 or 3 times that momentum.
- In the naïve coalescence model, the scaling is exact.
- This scaling is violated in the case of pions due to pion production in resonance decays of other hadrons.

Naïve quark coalescence model (I)

In the naïve coalescence model, the transverse momentum distribution of hadrons formed from partons is given by:

$$\frac{dN_m}{d^2\vec{p}} \propto \int \prod_{i=1}^{2} d^2\vec{p}_i \frac{dN_q}{d^2\vec{p}_i} \delta^{(2)}(\vec{p}_1-\vec{p}_2)\delta^{(2)}(\vec{p}-\vec{p}_1-\vec{p}_2)$$

$$\frac{dN_B}{d^2\vec{p}} \propto \int \prod_{i=1}^{3} d^2\vec{p}_i \frac{dN_q}{d^2\vec{p}_i} \delta^{(2)}(\vec{p}_1-\vec{p}_2)\delta^{(2)}(\vec{p}_2-\vec{p}_3)\delta^{(2)}(\vec{p}-\vec{p}_1-\vec{p}_2-\vec{p}_3)$$

The restrictions that this model places on the parton momenta are represented by the delta functions (quark momenta are equal and colinear).

Effect of parton momentum spread on meson elliptic flow

We define the total and relative momenta between two partons

$$p_1 + p_2 = p \quad \text{and} \quad k = (p_1 - p_2)/2$$

then $p_1 = \frac{p}{2} + k$ and $p_2 = \frac{p}{2} - k$

and assume collinear coalescence, i.e., $p_1 || p_2$

The meson momentum distribution is then given by

$$\frac{dN_M}{d^2p} \propto \frac{1}{N}\int_{b}^{a} dk\left(\frac{p^2}{4} - k^2\right)$$
$$\times [1 + 2v_{2,q}(p_1)v_{2,q}(p_2) + [2v_{2,q}(p_1) + 2v_{2,q}(p_2)]\cos(2\phi)]$$

Analytical Solution

Using Taylor expansions for quark v_2 and keeping only to second order terms in k, we are able to approximate the momentum distribution integral. The elliptic flow harmonic of the meson is given by the coefficient of 2cos(2φ) in the resulting expression, i.e.,

Case 1: $\Delta < p/2$

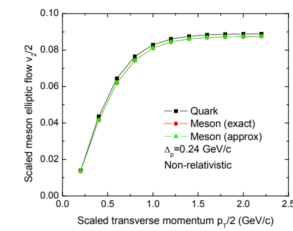
$$v_{2,M}(p) = \frac{2v_{2,q}(p/2) + \frac{1}{3}v''_{2,q}(p/2)\Delta^2}{1 + 2v^2_{2,q}(p/2) + \frac{2}{3}[v_{2,q}(p/2)v''_{2,q}(p/2) - v'^2_{2,q}(p/2)]\Delta^2}$$

Case 1: $\Delta > p/2$

$$v_{2,M}(p) = \frac{2v_{2,q}(p/2) + \frac{1}{5}v''_{2,q}(p/2)p^2}{1 + 2v^2_{2,q}(p/2) + \frac{2}{5}[v_{2,q}(p/2)v''_{2,q}(p/2) - v'^2_{2,q}(p/2)]p^2}$$

Meson elliptic flow from quark coalescence

$$v_{2,q}(p) = 0.089\tanh(1.89p - 0.22)$$



Effect of parton momentum spread on baryon elliptic flow (I)

Define total and relative momenta between three partons (Jacobi coordinates)

$$p = p_1 + p_2 + p_3$$
$$\rho = (p_1 - p_2)/\sqrt{2}$$
$$\lambda = (p_1 + p_2 - 2p_3)/\sqrt{6}$$

then

$$p_1 = \frac{p}{3} + \frac{\rho}{\sqrt{2}} + \frac{\lambda}{\sqrt{6}} \qquad p_2 = \frac{p}{3} - \frac{\rho}{\sqrt{2}} + \frac{\lambda}{\sqrt{6}}$$

$$p_3 = \frac{p}{3} - \frac{\sqrt{6}\lambda}{3}$$

Analytical Solution

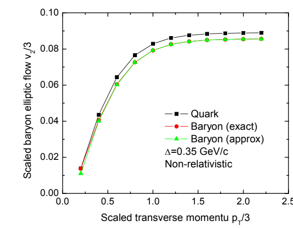
For the baryon case the integration is quite complicated and there are many cases; however, in the case of high momentum it is relatively easy to obtain the following results, in the same way that we obtained results in the meson case:

$$v_{2;b}(p) = \frac{N}{D}$$

$$N = 3v_{2,q}(p/3) + 3v^3_{2,q}(p/3) + \frac{1}{3}[v''_{2,q}(p/3) + 3v^2_{2,q}(p/3)v''_{2,q}(p) - 3v_{2,q}(p/3)v'^2_{2,q}(p/3)]\Delta^2$$

$$D = 1 + 2v^2_{2,q}(p/3) + \frac{2}{3}[2v_{2,q}(p/3)v''_{2,q}(p) - (v'_{2,q}(p/3))^2]\Delta^2$$

Baryon elliptic flow from quark coalescence



Relativistic Effects

In the center of mass frame of the partons, their relative momentum is:

$$p'_1 - p'_2 = \gamma(p_1 - p_2 - \beta(E_1 - E_2))$$

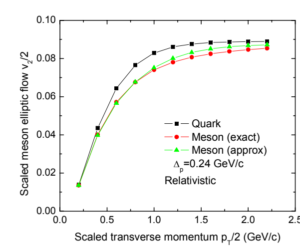
where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ and β is the velocity relative to the lab frame.

From this equation, the relative momentum in the lab frame can be approximated as:

$$p_1 - p_2 = (p'_1 - p'_2)\sqrt{1 + \frac{p^2}{4m}}$$

Which implies that the limit of relative momentum should be: $\Delta' = \Delta\sqrt{1 + \frac{p^2}{4m}}$

Meson elliptic flow from quark coalescence



Baryon elliptic flow from quark coalescence