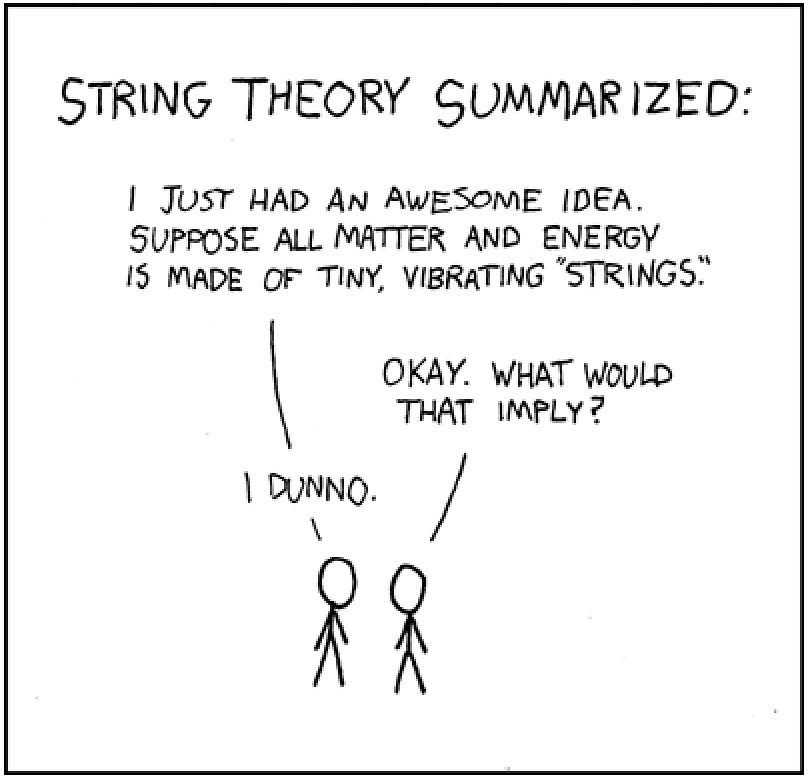
# String Theory:

## A Model Beyond Popular Physics





Kenny Wunder
Mississippi State University
TAMU Cyclotron

# Types of string theory

- Bosonic string theory
- Type I string theory
- Type II-A string theory
- Type II-B string theory
- Heterotic string theory

M-theory

# The big ideas *(in bosonic string theory)*

- On geometry…
  - Identification
  - Compactification
- D-branes
- Evolution of the action
- Parameterizing made easy

  *The light cone gauge*
- Becoming particles

# Some things about geometry: Identification

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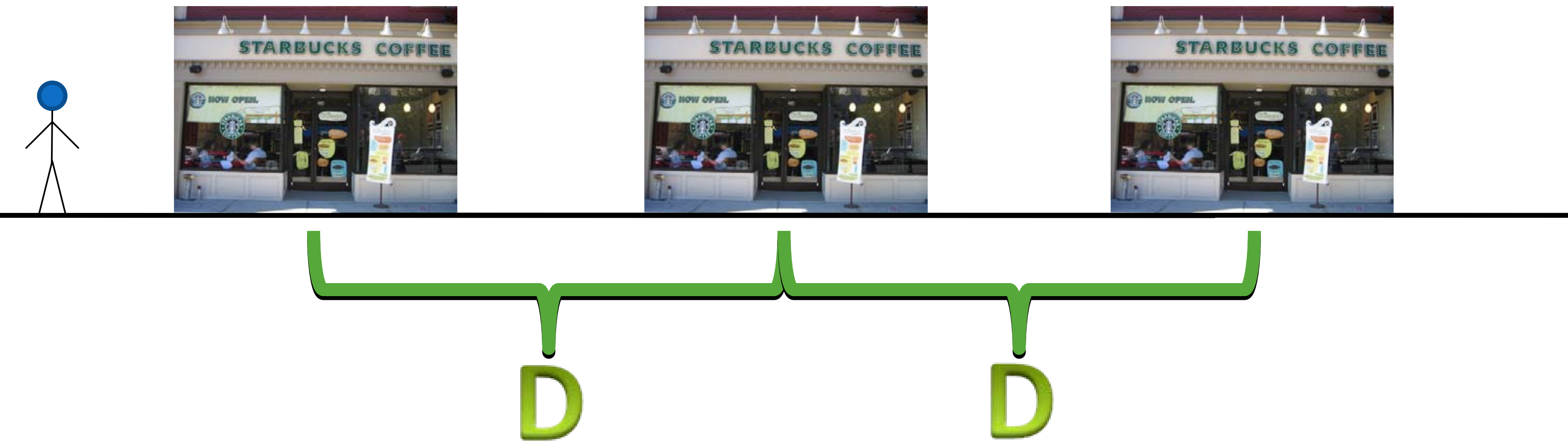
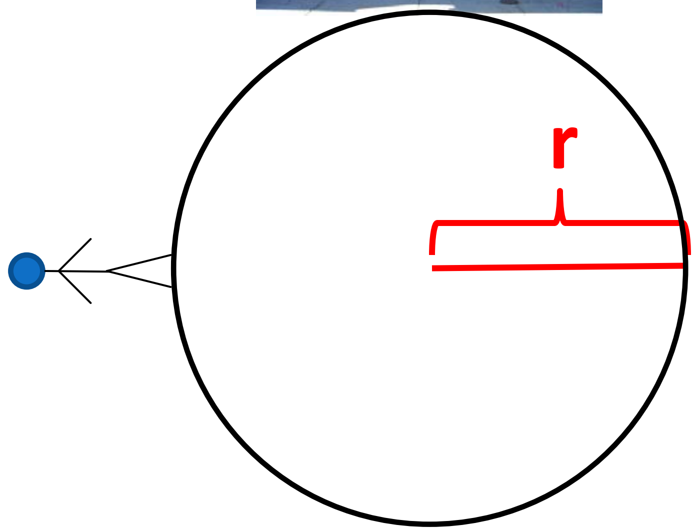
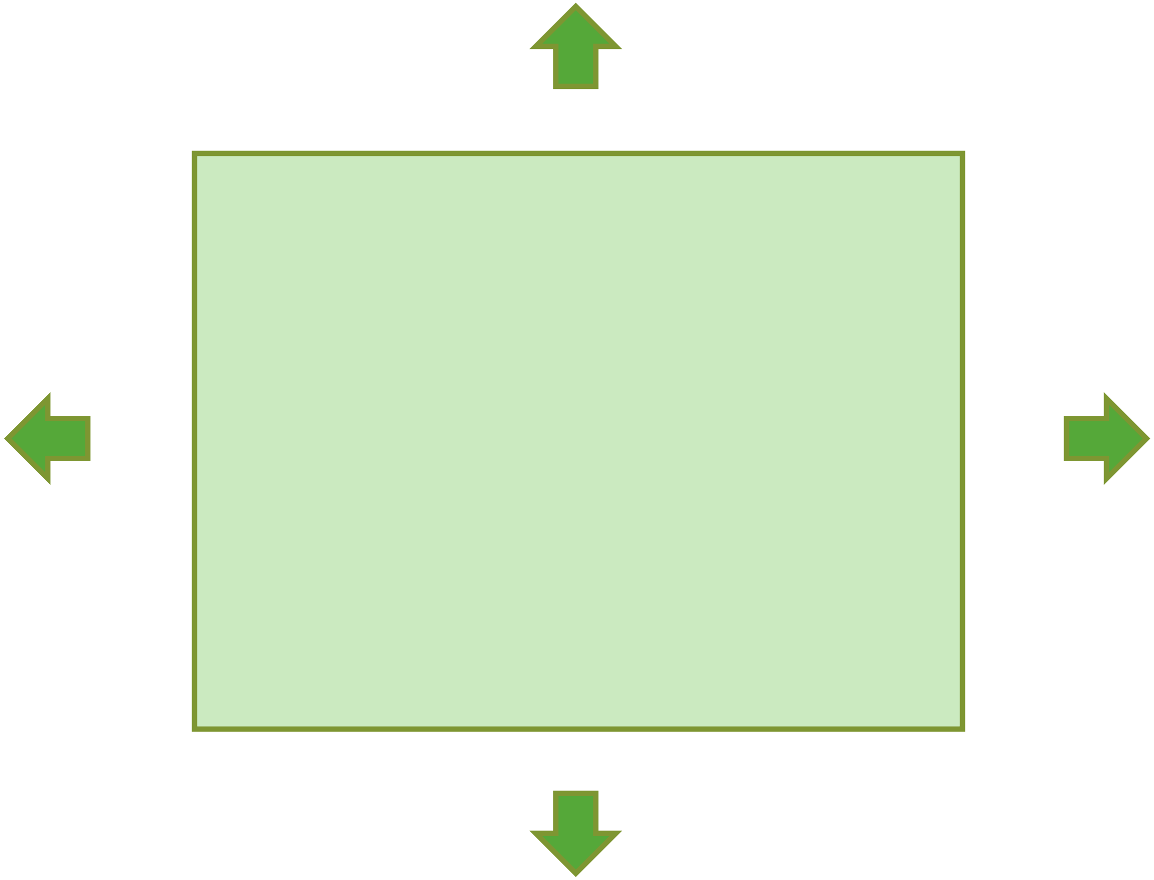


# Some things about geometry: Identification

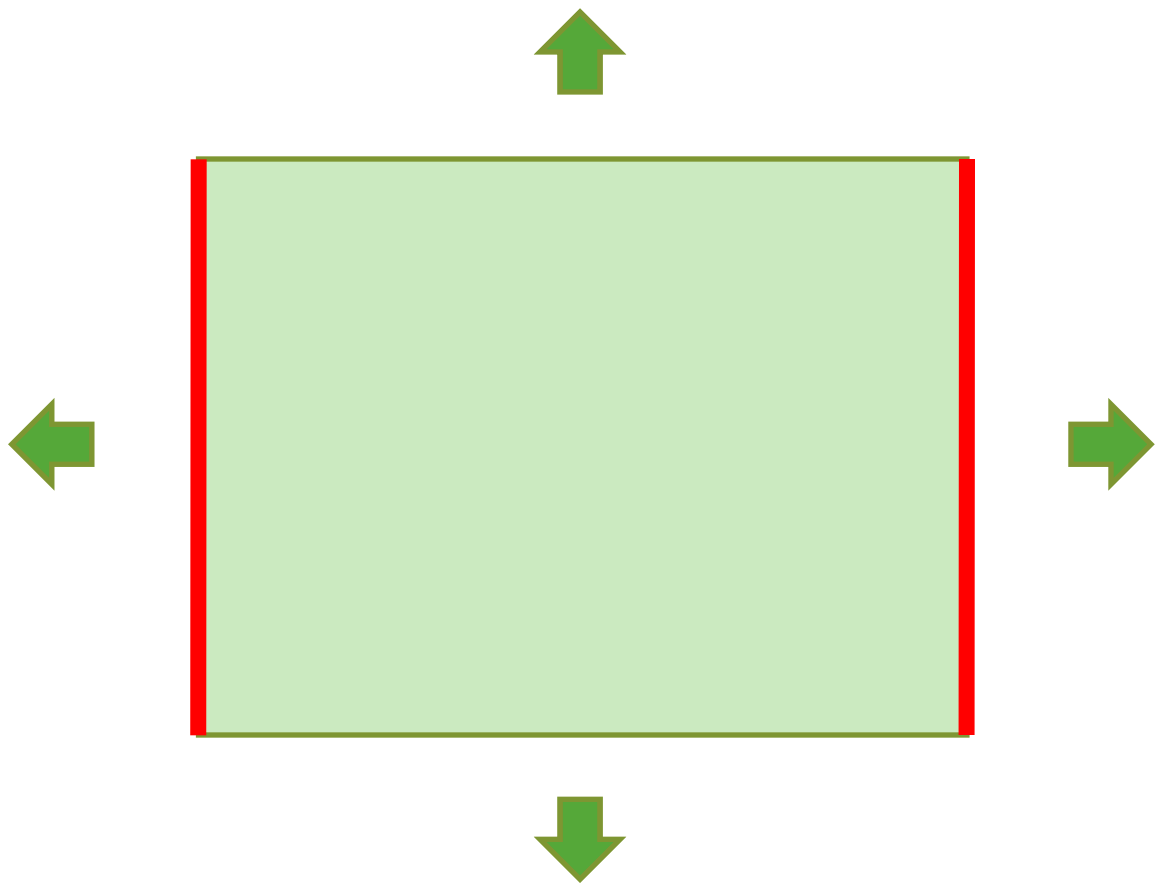




$$D = 2\pi r$$

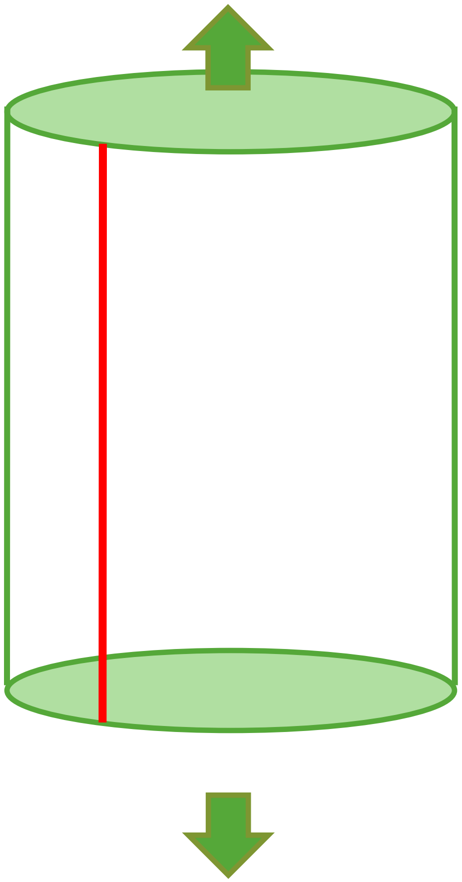
# Some things about geometry: Compactification



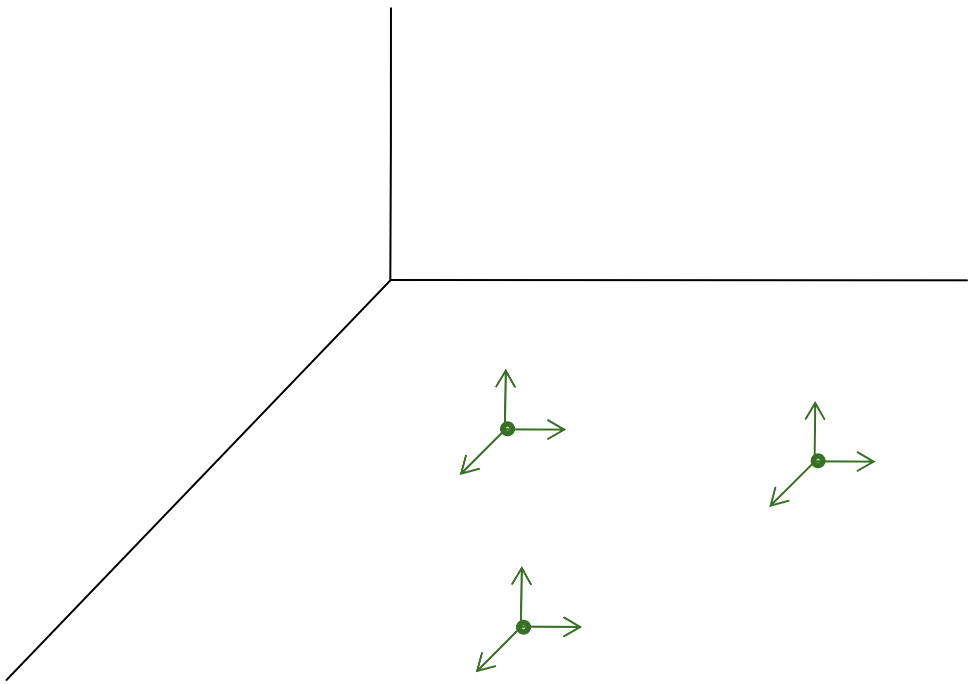
# Some things about geometry: Compactification



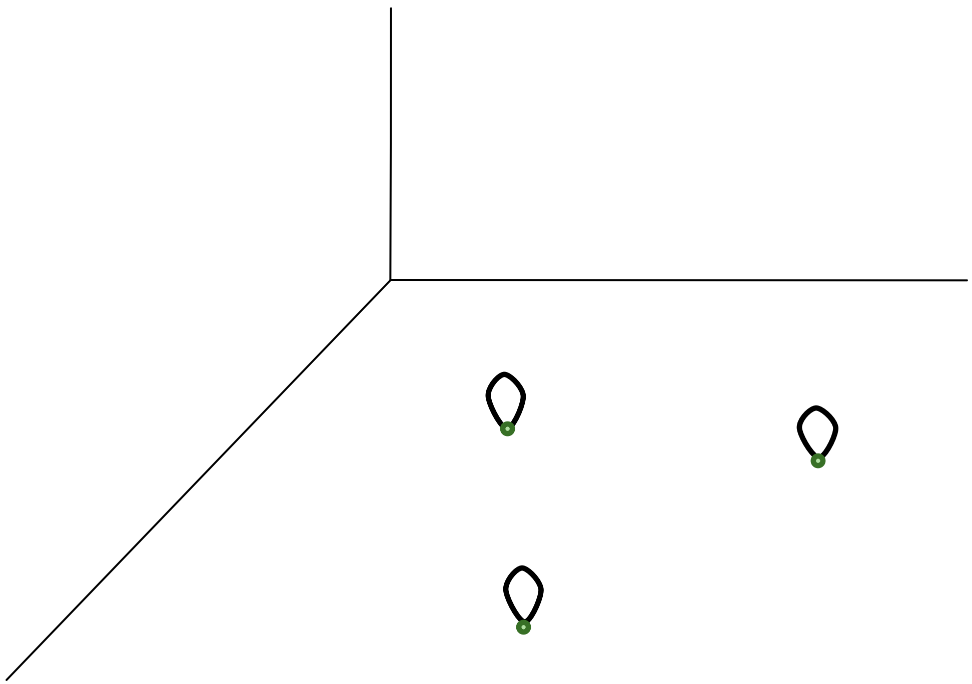
# Some things about geometry: Compactification



# Some things about geometry: Compactification



# Some things about geometry: Compactification



# Some things about geometry: Compactification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# Boundary conditions and D-branes

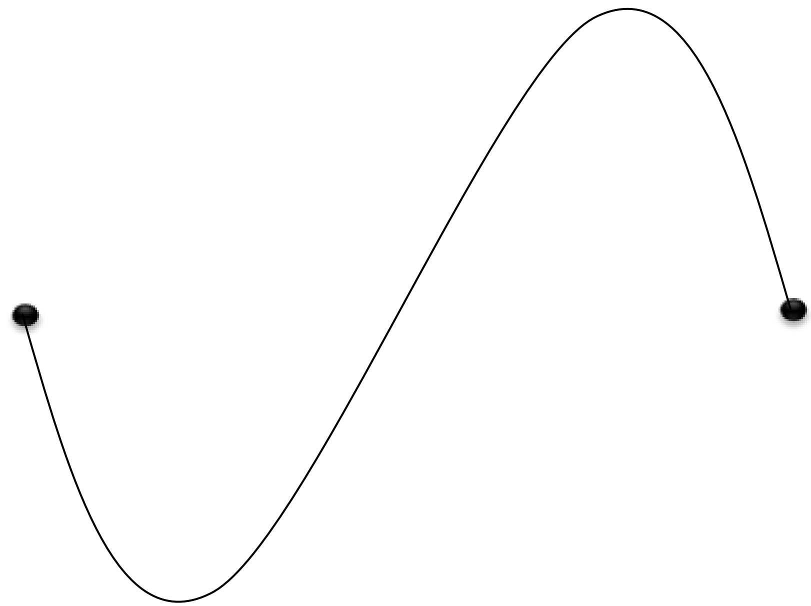
- Neumann boundary conditions

$$\frac{\partial y}{\partial x}(t,0) = 0 = \frac{\partial y}{\partial x}(t,a)$$

- Dirichlet boundary conditions

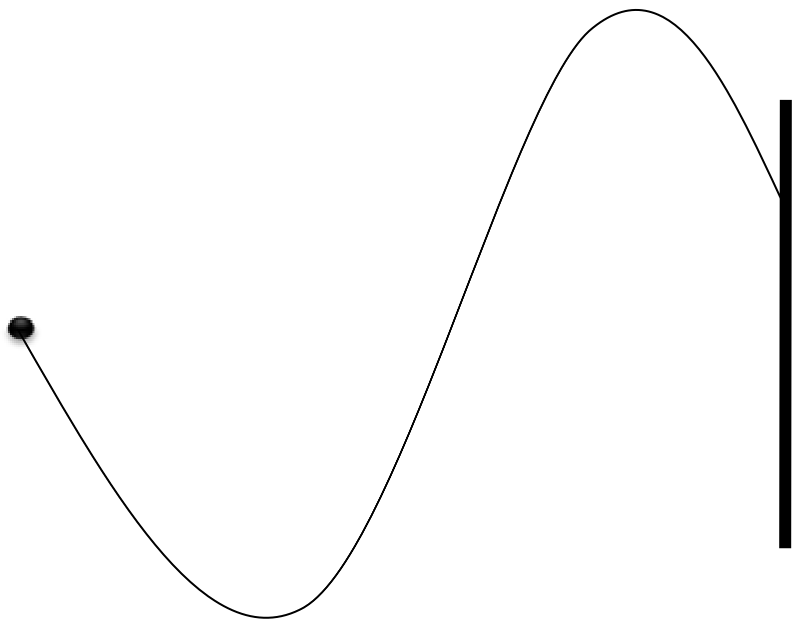
$$y(t,0) = 0 = y(t,a)$$

# Boundary conditions and D-branes



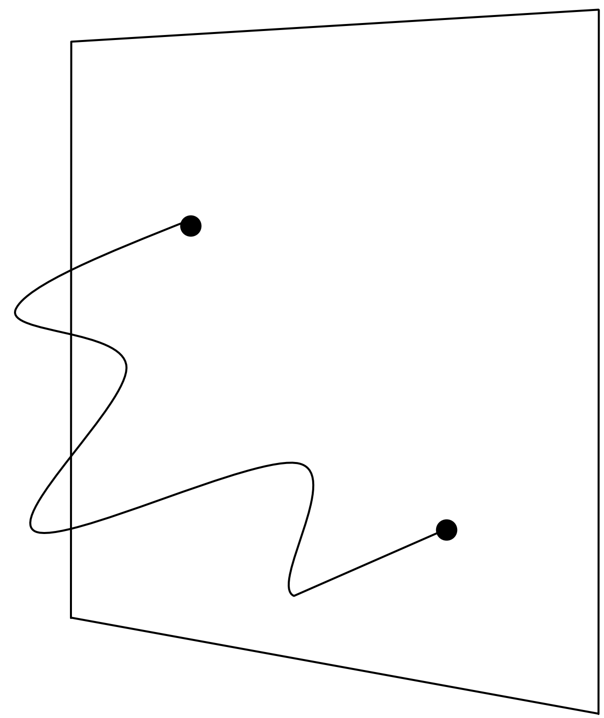
**D0-brane**

# Boundary conditions and D-branes



**D1-brane**

# Boundary conditions and D-branes



D2-brane

# The classical action
# The classical point particle

**Energy Considerations**

**+ Principle of Least Action**

$$\delta S = \int_{t_i}^{t_f} \delta x \left(-m\ddot{x}^2 - V'(x)\right) dt = 0$$

→ **F=ma**

# The classical action
# The classical string

$$L(t) = \int_0^a \left[ \frac{1}{2}(\mu_0)\left(\frac{\partial y}{\partial t}\right)^2 - \frac{1}{2} T_0 \left(\frac{\partial y}{\partial x}\right)^2 \right] dx$$

$$S = \int_{t_i}^{t_f} L(t)\, dt = \int_{t_i}^{t_f} dt \int_0^a dx \left[ \frac{1}{2}(\mu_0)\left(\frac{\partial y}{\partial t}\right)^2 - \frac{1}{2} T_0 \left(\frac{\partial y}{\partial x}\right)^2 \right]$$

$$\delta S = \int_{t_i}^{t_f} dt \int_0^a dx \left[ \frac{\partial}{\partial t}\left( \mu_0 \frac{\partial y}{\partial t} \delta y \right) - \mu_0 \frac{\partial^2 y}{\partial t^2} \delta y + \frac{\partial}{\partial x}\left( -T_0 \frac{\partial y}{\partial x} \delta y \right) + T_0 \frac{\partial^2 y}{\partial x^2} \delta y \right]$$

# The classical action
# The classical string

$$\mathcal{L}\left(\frac{\partial y}{\partial t},\frac{\partial y}{\partial x}\right)=\frac{1}{2}(\mu_0)\left(\frac{\partial y}{\partial t}\right)^2-\frac{1}{2}T_0\left(\frac{\partial y}{\partial x}\right)^2$$

$$\mathcal{P}^t\equiv\frac{\partial\mathcal{L}}{\partial\dot{y}}=\mu_0\frac{\partial y}{\partial t} \qquad \mathcal{P}^x\equiv\frac{\partial\mathcal{L}}{\partial y'}=-T_0\frac{\partial y}{\partial x}$$

# The classical action
# The classical string

$$\delta S = \int_{t_i}^{t_f} dt \int_0^a dx \left[ \frac{\partial}{\partial t}\left( \mu_0 \frac{\partial y}{\partial t} \delta y \right) - \mu_0 \frac{\partial^2 y}{\partial t^2} \delta y + \frac{\partial}{\partial x}\left( -T_0 \frac{\partial y}{\partial x} \delta y \right) + T_0 \frac{\partial^2 y}{\partial x^2} \delta y \right]$$

$$\delta S = \int_{t_i}^{t_f} dt \int_0^a dx \left[ \mathcal{P}^t \delta \dot{y} + \mathcal{P}^x \delta y' \right]$$

# The classical action
# The classical string

Again:
Energy Considerations
+ Principle of Least Action

$$\frac{\partial \mathcal{P}^t}{\partial t} \quad \frac{\partial \mathcal{P}^x}{\partial x}$$

the **wave equation**!!

# The relativistic action
# The relativistic point particle

Using Space-Time Distance

+ Energy Considerations

So,

Relativistic Action

# The relativistic action
# The relativistic point particle

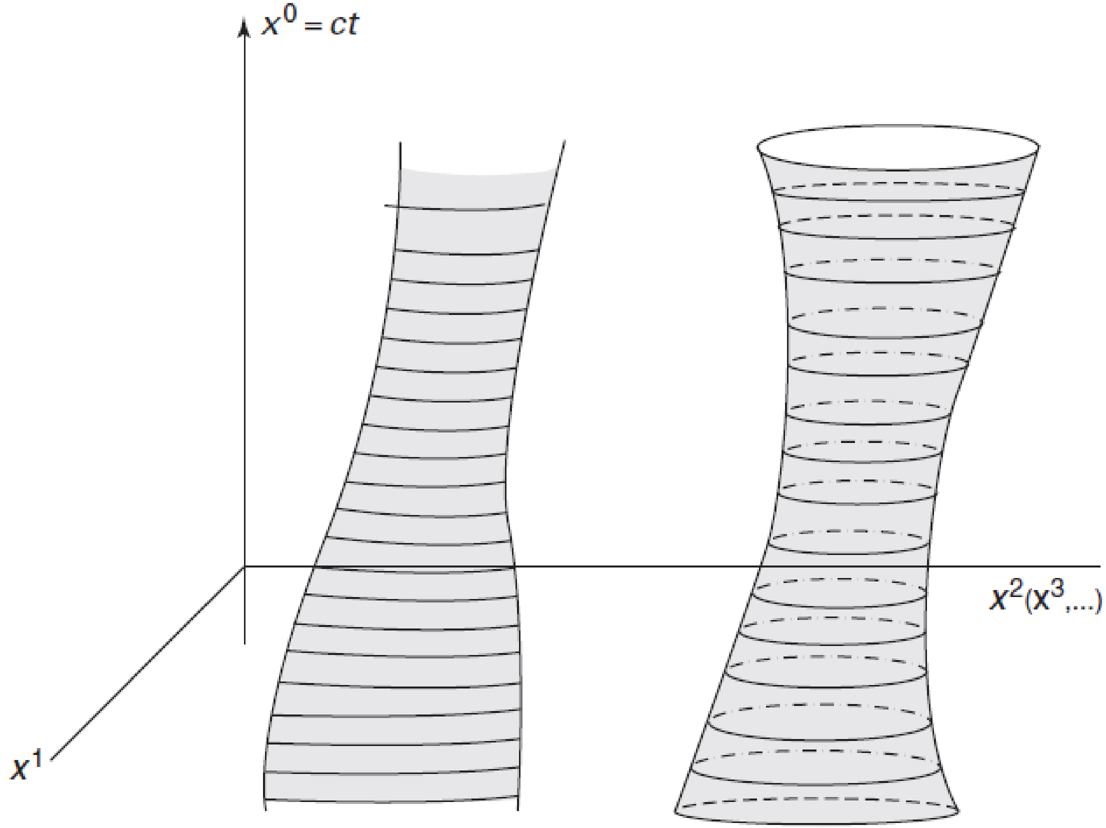
Beginning with the

Relativistic Measure

Changing Variables

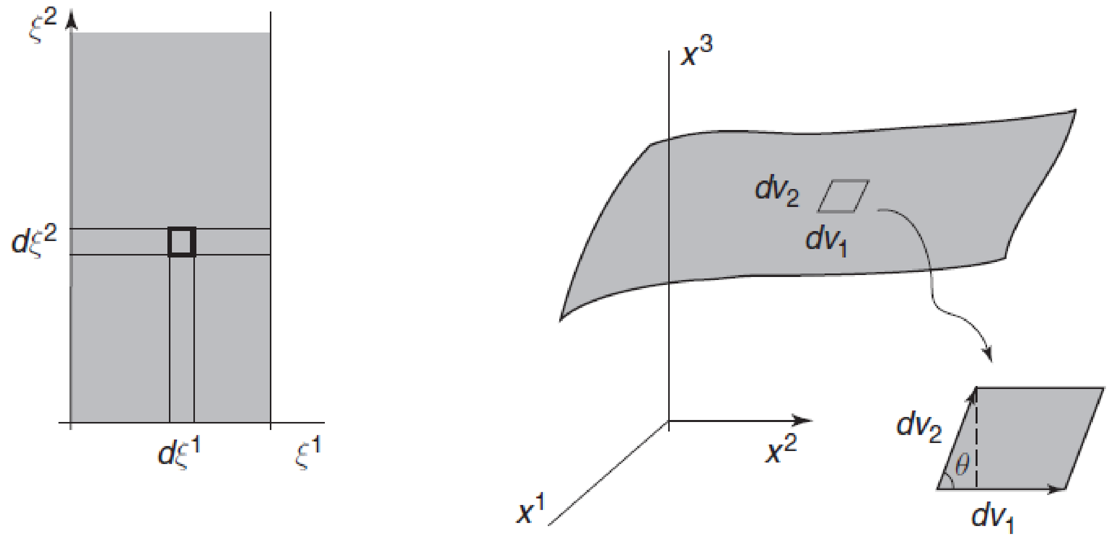
Lorentz Invariance

# The relativistic action
# The relativistic string




ref: Zwiebach, p.101.

# The relativistic action
# The relativistic string




ref: Zwiebach, p.102.

$$\vec{x}\left(\xi^1,\xi^2\right)=\left(x^1\left(\xi^1,\xi^2\right),x^2\left(\xi^1,\xi^2\right),x^3\left(\xi^1,\xi^2\right)\right)$$

$$d\vec{v}_1=\frac{\partial x}{\partial \xi^1}d\xi^1\ ,\ d\vec{v}_2=\frac{\partial x}{\partial \xi^2}d\xi^2$$

# The relativistic action
# The relativistic string

$$dA = |d\vec{v}_1||d\vec{v}_2||\sin\theta| = |d\vec{v}_1||d\vec{v}_2|\sqrt{1-\cos^2\theta}$$

$$= \sqrt{|d\vec{v}_1|^2|d\vec{v}_2|^2 - |d\vec{v}_1|^2|d\vec{v}_2|^2\cos^2\theta}$$

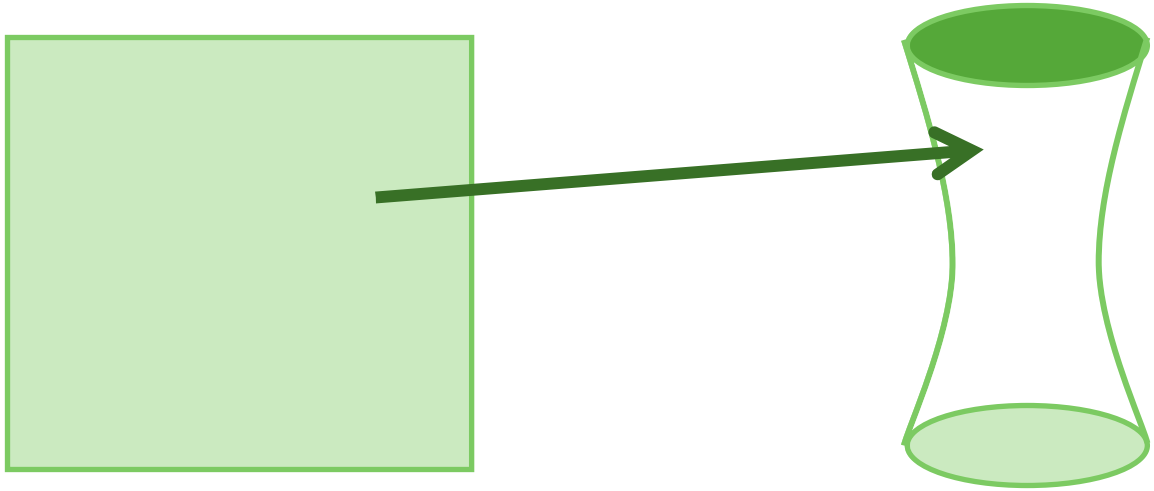
$$dA = \sqrt{(d\vec{v}_1 \cdot d\vec{v}_1)(d\vec{v}_2 \cdot d\vec{v}_2) - (d\vec{v}_1 \cdot d\vec{v}_2)^2}$$

# The relativistic action
# The relativistic string

$$dA = d\xi^1 d\xi^2 \sqrt{\left(\frac{\partial\vec{x}}{\partial\xi^1}\cdot\frac{\partial\vec{x}}{\partial\xi^1}\right)\left(\frac{\partial\vec{x}}{\partial\xi^2}\cdot\frac{\partial\vec{x}}{\partial\xi^2}\right)-\left(\frac{\partial\vec{x}}{\partial\xi^1}\cdot\frac{\partial\vec{x}}{\partial\xi^2}\right)^2}$$

$$A = \int d\xi^1 d\xi^2 \sqrt{\left(\frac{\partial\vec{x}}{\partial\xi^1}\cdot\frac{\partial\vec{x}}{\partial\xi^1}\right)\left(\frac{\partial\vec{x}}{\partial\xi^2}\cdot\frac{\partial\vec{x}}{\partial\xi^2}\right)-\left(\frac{\partial\vec{x}}{\partial\xi^1}\cdot\frac{\partial\vec{x}}{\partial\xi^2}\right)^2}$$

# The relativistic action
# The relativistic string



$$x^{\mu} = \left(x^0, x^1, \ldots, x^d\right)$$

$$(\tau, \sigma) \rightarrow \left(X^0(\tau,\sigma), X^1(\tau,\sigma), \ldots, X^d(\tau,\sigma)\right)$$

# The relativistic action
# The relativistic string

$$A = \int d\tau\, d\sigma \sqrt{\left(\frac{\partial X}{\partial \tau}\cdot\frac{\partial X}{\partial \tau}\right)\left(\frac{\partial X}{\partial \sigma}\cdot\frac{\partial X}{\partial \sigma}\right) - \left(\frac{\partial X}{\partial \tau}\cdot\frac{\partial X}{\partial \sigma}\right)^2}$$

$$A = \int d\tau\, d\sigma \sqrt{\left(\frac{\partial X}{\partial \tau}\right)^2\left(\frac{\partial X}{\partial \sigma}\right)^2 - \left(\frac{\partial X}{\partial \tau}\cdot\frac{\partial X}{\partial \sigma}\right)^2}$$

# The relativistic action
# The relativistic string

**Worldline → Worldsheet**

**dS → dA**

$$S = -\frac{T_0}{c}\int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \sqrt{\left(\dot{X}\cdot X'\right)^2 - \left(\dot{X}\right)^2\left(X'\right)^2}$$

**Relativistic String Action Defined in Area on the Sheet**

# The relativistic action
# The relativistic string

$$\mathcal{L}\left(\dot{X}^{\mu},(X')^{\mu}\right)=-\frac{T_0}{c}\sqrt{\left(\dot{X}\cdot X'\right)^2-\left(\dot{X}\right)^2\left(X'\right)^2}$$

$$\mathcal{P}^{\tau}_{\mu}\equiv\frac{\partial\mathcal{L}}{\partial\dot{X}^{\mu}}=-\frac{T_0}{c}\frac{\left(\dot{X}\cdot X'\right)X'_{\mu}-\left(X'\right)^2\dot{X}_{\mu}}{\sqrt{\left(\dot{X}\cdot X'\right)^2-\left(\dot{X}\right)^2\left(X'\right)^2}}$$

$$\mathcal{P}^{\sigma}_{\mu}\equiv\frac{\partial\mathcal{L}}{\partial X^{\mu\prime}}=-\frac{T_0}{c}\frac{\left(\dot{X}\cdot X'\right)\dot{X}_{\mu}-\left(\dot{X}\right)^2X'_{\mu}}{\sqrt{\left(\dot{X}\cdot X'\right)^2-\left(\dot{X}\right)^2\left(X'\right)^2}}$$

# The relativistic action
# The relativistic string

$$\delta S = 0 \quad \rightarrow \quad \frac{\partial \mathcal{P}^{\tau}_{\mu}}{\partial \tau} + \frac{\partial \mathcal{P}^{\sigma}_{\mu}}{\partial \sigma} = 0$$

$$\frac{T_0}{c} = \frac{1}{2\pi\alpha'} \qquad l_s = \sqrt{\alpha'}$$

# The relativistic action
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$$l_s = \sqrt{\alpha'}$$

# Effective parameterization:
# The light cone gauge

$$\left(n \cdot p\right)\sigma = \frac{2\pi}{\beta}\int_0^{\sigma} d\tilde{\sigma}\, n \cdot \mathcal{P}^{\tau}(\tau, \tilde{\sigma}) \quad : \sigma - gauge$$

$$n \cdot X(\tau, \sigma) = \beta\alpha'\left(n \cdot p\right)\tau \quad : \tau - gauge$$

$$\dot{X} \cdot X' = 0$$

$$\dot{X}^2 + X'^2 = 0$$

# Effective parameterization: The light cone gauge

$$\mathcal{P}^\tau_\mu = \frac{1}{2\pi\alpha'}\dot{X}_\mu \qquad \mathcal{P}^\sigma_\mu = -\frac{1}{2\pi\alpha'}X'_\mu$$

**A bunch of fancy math stuff…**

$$\frac{\partial \mathcal{P}^\tau_\mu}{\partial \tau} + \frac{\partial \mathcal{P}^\sigma_\mu}{\partial \sigma} = 0 \quad \rightarrow \quad \boxed{\ddot{X}^\mu - X''^\mu = 0}$$

**The Wave Equation… AGAIN!!**

# Solving the wave equation

**Where is the string going?**

$$X^{\mu}(\tau,\sigma) = x_0^{\mu} + \sqrt{2\alpha'}\alpha_0^{\mu}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{\mu}e^{-in\tau}\cos n\sigma$$

**How is it oscillating?**

# Becoming particles

The string energy stored

$$M^2 = \frac{1}{\alpha'} \sum_{n=1}^{\infty} n a_n^{I*} a_n^I$$

translates into particle mass

# Closed stings and open strings

Closed strings → Gravitons

Open strings → Other bosons

# Acknowledgements

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# Acknowledgements

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Cyclotron Institute

Texas A&M University

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# The relativistic action
# The relativistic point particle

$$\delta S = -\int_{\tau_i}^{\tau_f} d\tau \, \delta x^{\mu}(\tau) \frac{dp_{\mu}}{d\tau}$$

$$\delta S = 0 \quad \rightarrow \quad \frac{dp_{\mu}}{d\tau} = 0$$

$$\frac{dp_{\mu}}{d\tau} = \frac{d}{d\tau}\left(p_{\mu}\right) = \frac{d}{d\tau}\left(\eta_{\mu\nu} p^{\nu}\right) = \eta_{\mu\nu} \frac{d}{d\tau}\left(p^{\nu}\right)$$

$$p^{\mu} = mc \frac{dx^{\mu}}{ds} \quad \rightarrow \quad \frac{d^2 x^{\mu}}{ds^2} = 0$$